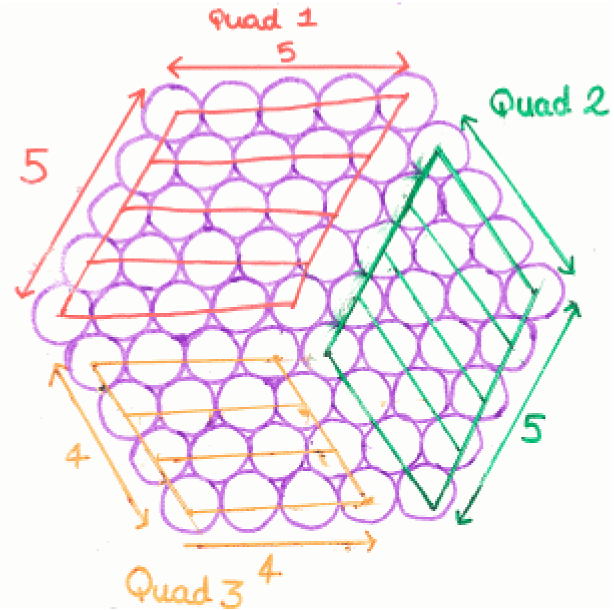
Steel Cables

Here is some work done on the problem ‘Steel Cables’ by Group 1. Can you explain their reasoning?



Size	Quad 1	Quad 2	Quad3	T
2	2×2	2×1	1×1	7
3	3×3	3×2	2×2	19
4	4×4	4×3	3×3	37
5	5×5	4×5	4×4	61
6	6×6	6×5	5×5	91
10	10×10	10×9	9×9	271
n	$n \times n$	$n \times (n-1)$	$(n-1)^2$	?
	↑ n^2	↑ n^2-n	↑ n^2-2n+1	

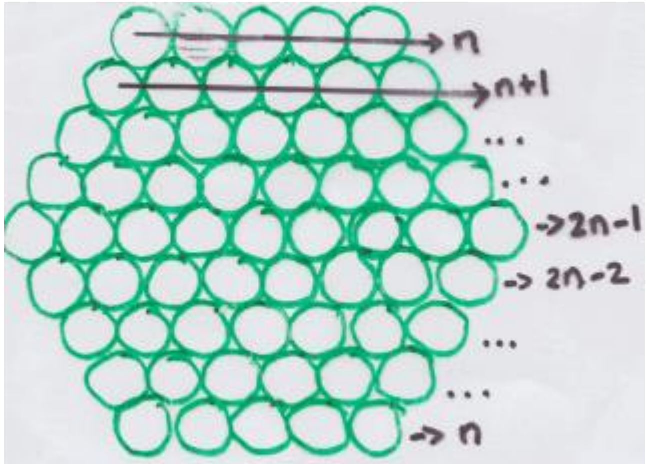
$$T = n^2 + n^2 - n + n^2 - 2n + 1$$

$$\Rightarrow 3n^2 - 3n + 1$$

or

$$3n(n-1) + 1$$

Here is some work done on the problem ‘Steel Cables’ by Group 2. Can you explain their reasoning?



To find the sum you must add up all the rows:

$n + 2n-1 = 3n-1$

$n+1 + 2n-2 = 3n-1$

$n+2+2n-3 = 3n-1$

$\vdots \quad \vdots \quad = \text{'' ''}$

$2n-1 + n = 3n-1$

$3n-1$ is the sum of two of the rows so there are n pairs of $3n-1$:

$n(3n-1)$

BUT we repeated $2n-1$. When there is only one of this, so take away the extra $2n-1$

$n(3n-1) - 2n-1$

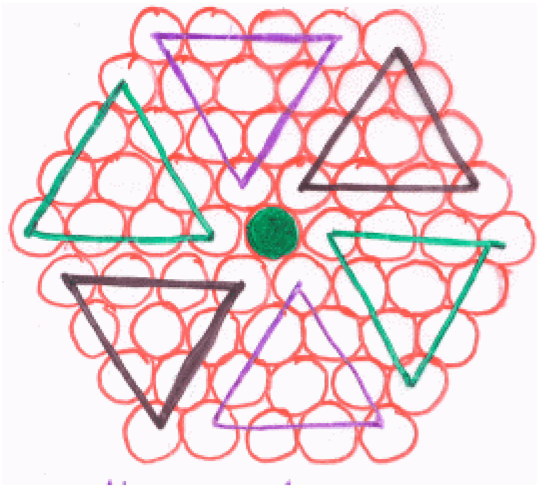
Simplify it:

$= 3n^2 - n - (2n-1)$

$= 3n^2 - n - 2n + 1$

$= 3n(n-1) + 1$

Here is some work done on the problem 'Steel Cables' by Group 3. Can you explain their reasoning?



We noticed there are always 6 triangles in a hexagon - all equal

The areas of these triangles are triangular numbers

The formula for triangular no = $\frac{n(n+1)}{2}$

However, the side of one of these triangles is not 'n' but one less so the length of the side of the triangal) $n-1$

Substitute this into the formula instead of n :

$$\frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$$

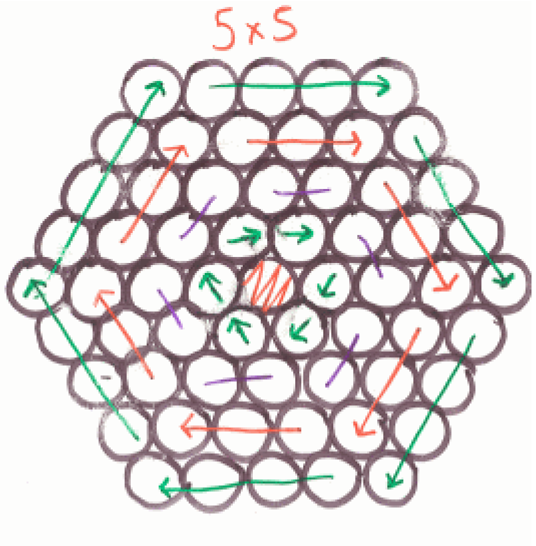
We know there are always 6 triangles in a hexagon so the total area will be

6 x triangle area + the extra one in the centre

$$6\left(\frac{n(n-1)}{2}\right)+1$$

$$\cancel{6}\,3\left(\frac{n(n-1)}{\cancel{2}\,1}\right)+1 = 3n(n-1)+1$$

Here is some work done on the problem 'Steel Cables' by Group 4. Can you explain their reasoning?



1

6×1

6×2

6×3

6×4

...

$6(n-1)$

We noticed that the area of each ring followed this pattern. To find the total we needed to add the areas of each ring

$$1 + 6 \times 1 + 6 \times 2 + 6 \times 3 \ldots 6(n-1) =$$

$$1 + 6(1 + 2 + 3 + 4 \ldots n-1)$$

$$1 + 6\left(\frac{n(n-1)}{2}\right)$$

$$\boxed{1 + 3n(n-1)}$$